

Roles of non-equilibrium conduction electrons on magnetization dynamics of ferromagnets

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Abstract

The mutual dependence of spin-dependent conduction and magnetization dynamics of ferromagnets provides the key mechanisms in various spin-dependent phenomena. We compute the response of the conduction electron spins to a spatial and time varying magnetization $\mathbf{M}(\mathbf{r}, t)$ within the time-dependent semiclassical transport theory. We show that the induced non-equilibrium conduction spin density in turn generates four spin torques acting on the magnetization—with each torque playing different roles in magnetization dynamics. By comparing with recent theoretical models, we find that one of these torques that has not been previously identified is crucial to consistently interpret experimental data on domain wall motion.

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Recently, there are emerging interests in the interplay between spin-dependent transport properties and magnetization dynamics of ferromagnets. Giant magnetoresistive effect in magnetic multilayers [1] is one of the examples that the spin-transport is controlled by magnetization dynamics (or configurations). Spin angular momentum transfer [2], or spin torque, manifests the magnetization dynamics controlled by spin-polarized conduction electrons. There are quite a few closely related phenomena reported recently, e.g., enhancement of damping parameters due to spin pumping [3, 4] and reaction spin torques [5], dynamic RKKY interaction [6], spin echo [7] and adiabatic spin torques in a domain wall [8]. These proposed or observed phenomena motivated us to look for a theoretical framework which is capable to address the above phenomena on an equal footing. The essence of the above phenomena is to recognize two types of electrons: spin-dependent transport is provided by electrons at the Fermi level and magnetization dynamics may involve electrons below the Fermi sea. While it is impossible to unambiguously separate electrons of transport from electrons of magnetization in a real ferromagnet, it has been conventionally modeled via a “s-d” Hamiltonian,

$$H_{sd} = -J_{ex} \mathbf{s} \cdot \mathbf{S} \quad (1)$$

where \mathbf{s} and \mathbf{S} are the spins of itinerant and localized electrons, and J_{ex} is the exchange coupling strength. In this letter, we show that the above simple s-d model in fact captures most of the physics on the interplay between spin-polarized transport of itinerant electrons and magnetization dynamics of local moments. We will first derive a linear response function for the conduction electron spin in the presence of a time and spatially varying local moment, and then by using the same s-d model to calculate the spin torque on the magnetization dynamics as a result of the induced non-equilibrium conduction electron spin. Among other things, we have found four distinct spin torques on the magnetization. Three of them are closely related to previously derived torques by using different methods. One of the derived torque is new; it describes the mis-tracking between the conduction electron spin and the spatially varying local moment. We further show that our formulation can be conveniently applied to study magnetization dynamics. An example of domain wall motion is illustrated in the end of the paper.

The dynamics of the conduction electron will be considered separately from that of local magnetization. We treat the itinerant spin \mathbf{s} as a full quantum mechanical operator whose equation of motion is governed by a transport equation, but we approximate \mathbf{S} as a classical

magnetization vector whose dynamics is much slower than that of itinerant spins, i.e., we replace \mathbf{S} by a classical magnetization $\mathbf{M}(\mathbf{r}, t)$

$$H_{sd} = -\frac{J_{ex}}{\hbar M_s} \mathbf{s} \cdot \mathbf{M}(\mathbf{r}, t) \quad (2)$$

where $|\mathbf{M}(\mathbf{r}, t)| = M_s$ is the saturation magnetization. We first determine the induced spin density for a given $\mathbf{M}(\mathbf{r}, t)$ and then derive the reaction of the induced spin density to the magnetization.

In the present study, the non-equilibrium conduction electrons are generated by applying either a DC electric field or a time-dependent magnetic field. While the electric field directly generates the charge and spin currents in conducting ferromagnets, the time-dependent magnetic field is to drive the magnetization motion that induces a non-equilibrium spin density via “s-d” interaction. The conduction electron spin operator satisfies the generalized spin continuity equation,

$$\frac{\partial \mathbf{s}}{\partial t} + \nabla \cdot \hat{\mathcal{J}} = \frac{1}{i\hbar} [\mathbf{s}, H_{sd}] - \Gamma_{re}(\mathbf{s}) \quad (3)$$

where $\hat{\mathcal{J}}$ is the spin current operator, and $\Gamma_{re}(\mathbf{s})$ represents the spin relaxation due to scattering with impurities, electrons, etc. By defining electron spin density $\mathbf{m}(\mathbf{r}, t) = \langle \mathbf{s} \rangle$ and spin current density $\mathcal{J}(\mathbf{r}, t) = \langle \hat{\mathcal{J}} \rangle$ where $\langle \rangle$ represents the average over all occupied electronic states, e.g., $\langle \mathbf{s} \rangle = \text{Tr}(\rho \mathbf{s})$ where the trace is over all electronic as well as spin states, and ρ is the density operator, one obtains a semiclassical Bloch equation for the conduction electron spin density,

$$\frac{\partial \mathbf{m}}{\partial t} + \nabla \cdot \mathcal{J} = -\frac{1}{\tau_{ex} M_s} \mathbf{m} \times \mathbf{M}(\mathbf{r}, t) - \langle \Gamma(\mathbf{s}) \rangle \quad (4)$$

where the commutator in Eq. (3) has been explicitly calculated by utilizing Eq. (2), and we have defined $\tau_{ex} = \hbar/J_{ex}$.

Next, we separate the induced spin density \mathbf{m} into two terms,

$$\mathbf{m}(\mathbf{r}, t) = \mathbf{m}_0(\mathbf{r}, t) + \delta \mathbf{m}(\mathbf{r}, t) = n_0 \frac{\mathbf{M}(\mathbf{r}, t)}{M_s} + \delta \mathbf{m}(\mathbf{r}, t) \quad (5)$$

where n_0 is the local equilibrium spin density whose direction is parallel to the magnetization. The first term in Eq. (5) represents the equilibrium spin density when the conduction electron spin relaxes to its equilibrium value at an instantaneous time t . Since the dynamics of the magnetization is slow compared to that of conduction electrons, it is reasonable to assume

the spin of the conduction electrons approximately follows the direction of the local moment, known as the adiabatic process. The second term represents the deviation from this adiabatic process. Similarly, we write the spin current density as

$$\mathcal{J}(\mathbf{r}, t) = \mathcal{J}_0(\mathbf{r}, t) + \delta\mathcal{J}(\mathbf{r}, t) = -(\mu_B P/e)\mathbf{j}_e \otimes \frac{\mathbf{M}(\mathbf{r}, t)}{M_s} + \delta\mathcal{J}(\mathbf{r}, t) \quad (6)$$

where e is the electron charge, j_e is the current density, μ_B is the Bohr magneton, and P is the spin current polarization of the ferromagnet. Note that the spin current is a tensor that consists of two vectors: the charge current and the spin polarization of the current. The first term in Eq. (6) is the spin current whose spin polarization is parallel to the local magnetization $\mathbf{M}(\mathbf{r}, t)$. To solve for the non-equilibrium spin density in a closed form, we consider the following simplifications. First, we use a simple relaxation time approximation to model the relaxation term in Eq. (4), i.e., we write $\langle \Gamma(\mathbf{s}) \rangle = \delta\mathbf{m}(\mathbf{r}, t)/\tau_{sf}$ where τ_{sf} is the spin-flip relaxation time. The approximation is necessary in order to obtain a simple analytic expression. Second, we only consider the linear response of $\delta\mathbf{m}$ to the electric current j_e and to the time derivative of magnetization $\partial\mathbf{M}/\partial t$. Since $\delta\mathbf{m}$ is already the first order, $\partial\delta\mathbf{m}/\partial t$ will be the order of $j_e \cdot \partial\mathbf{M}/\partial t$ or $\partial^2\mathbf{M}/\partial t^2$ and thus it can be discarded. Within the semiclassical picture of the transport, the non-adiabatic current density $\delta\mathcal{J}$ is related to the non-equilibrium spin density $\delta\mathbf{m}$ via $\delta\mathcal{J} = -D_0\nabla\delta\mathbf{m}$ where D_0 is the diffusion constant. By inserting Eqs. (5) and (6) into (4) and utilizing the above simplification, we obtain the closed form for the non-equilibrium spin density

$$D_0\nabla^2\delta\mathbf{m} - \frac{1}{\tau_{ex}M_s}\delta\mathbf{m} \times \mathbf{M} - \frac{\delta\mathbf{m}}{\tau_{sf}} = \frac{n_0}{M_s} \cdot \frac{\partial\mathbf{M}}{\partial t} - \frac{\mu_BP}{eM_s}(\mathbf{j}_e \cdot \nabla)\mathbf{M} \quad (7)$$

One immediately realizes that the non-equilibrium spin density is created by two source terms on the right side of Eq. (7): one is the time variation and the other is the spatial variation of the magnetization. The solution of the above differential equation depends on the detail structure of the magnetization vector. Here we assume that the magnetization varies slowly in space, i.e., the domain wall width W of the magnetization is much larger than the transport length scale defined in the footnote [9]. In this case, the spatial derivation, the first term in Eq. (7), can be discarded [9]. Then Eq. (7) becomes a simple vector algebraic equation and by using the elementary vector manipulation we readily obtain an explicit expression for the non-equilibrium spin density

$$\delta\mathbf{m} = \frac{\tau_{ex}}{(1+\xi^2)} \left(-\frac{\xi n_0}{M_s} \frac{\partial\mathbf{M}}{\partial t} - \frac{n_0}{M_s^2} \mathbf{M} \times \frac{\partial\mathbf{M}}{\partial t} + \frac{\mu_BP\xi}{eM_s} (\mathbf{j}_e \cdot \nabla)\mathbf{M} + \frac{\mu_BP}{eM_s^2} \mathbf{M} \times (\mathbf{j}_e \cdot \nabla)\mathbf{M} \right) \quad (8)$$

where $\xi = \tau_{ex}/\tau_{sf}$. The above induced spin density in turn exerts a spin torque on the magnetization. From Eq. (2), the torque is $\mathbf{T} = -(J_{ex}/\hbar M_s)\mathbf{M} \times \mathbf{m} = -(J_{ex}/\hbar M_s)\mathbf{M} \times \delta\mathbf{m}$. By using Eq. (8), we have

$$\mathbf{T} = \frac{1}{1 + \xi^2} \left(-\frac{n_0}{M_s} \frac{\partial \mathbf{M}}{\partial t} + \frac{\xi n_0}{M_s^2} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} - \frac{\mu_B P}{e M_s^3} \mathbf{M} \times [\mathbf{M} \times (\mathbf{j}_e \cdot \nabla) \mathbf{M}] - \frac{\mu_B P \xi}{e M_s^2} \mathbf{M} \times (\mathbf{j}_e \cdot \nabla) \mathbf{M} \right) \quad (9)$$

There are four terms; the first two are from magnetization variation in time and the last two in space. Interestingly, the first two terms are independent of the current. The last two terms represent the current-driven effect since they are proportional to the current. We now discuss the role of each spin torque below.

The standard Landau-Lifshitz-Gilbert (LLG) equation consists of a precessional term due to an effective field and a phenomenological damping term. In addition to these two torques, the above torque \mathbf{T} is now added to the LLG equation,

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{eff} + \frac{\alpha}{M_s} \left(\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} \right) + \mathbf{T} \quad (10)$$

where γ is the gyromagnetic ratio, \mathbf{H}_{eff} is the effective magnetic field, α is the Gilbert damping parameter. We immediately realize that the first term in Eq. (9) is simply to renormalize the gyromagnetic ratio while the second term is to renormalize the damping parameter. Thus if we introduce an effective gyromagnetic ratio γ' and the damping parameter α' ,

$$\gamma' = \gamma(1 + \eta)^{-1}; \quad \gamma' \alpha' = \gamma(\alpha + \xi \eta)$$

where we have defined $\eta = (n_0/M_s)/(1 + \xi^2)$, LLG equation remains in the same form. We point out that the modification of the gyromagnetic ratio and the damping parameter through the present mechanism is rather small in transition metal ferromagnets. For a typical ferromagnet (Ni, Co, Fe and their alloys), $J_{ex} \approx 1$ eV, $\tau_{sf} \approx 10^{-12}$ s, $n_0/M_s \approx 10^{-2}$, $\xi \approx 10^{-2}$ and thus η is about 10^{-2} and $\xi \eta$ is of the order of 10^{-4} —much smaller than the typical damping parameter of the order of 10^{-2} . Therefore, we conclude that the temporal spin torque driven by the exchange interaction only slightly modifies the damping parameter and can not be identified as a leading mechanism for magnetization damping.

At this point, we should compare other theories on the spin torque. Tserkovnyak *et al.* [3, 7] proposed an adiabatic spin pumping mechanism to explain the enhancement of Gilbert damping parameters. Ho *et al* suggested a radiation field induced by magnetization

precessional motion of magnets [5]. Most recently, a similar s-d model in the presence of the time-dependent magnetization has been considered [10]. The present approach reduces to these theories in the simple limit considered for these two terms. In fact, the idea of this temporal spin torque had been suggested earlier: when the magnetization varies in time, the spin of the conduction electrons tends to follow the direction of the magnetization with a time delay given by spin relaxation time; this phenomenon was named as “breathing Fermi surface” [11]. We are now able to consider this physics of the enhanced damping on the equal footing as the current induced spin torques.

Our main focus here is the spin torque due to the spatially non-uniform magnetization vector, the last two terms in Eq. (9). Since the temporal spin torques can be completely absorbed by the re-definition of the gyromagnetic ratio and damping constant, we should now just ignore them and concentrate on the role of spin torque generated by the non-uniform magnetization. We thus write the full equation for the magnetization dynamics below

$$\frac{\partial \mathbf{M}}{\partial t} = -\gamma \mathbf{M} \times \mathbf{H}_{eff} + \frac{\alpha}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial t} - \frac{b_J}{M_s^2} \mathbf{M} \times \left(\mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x} \right) - \frac{c_J}{M_s} \mathbf{M} \times \frac{\partial \mathbf{M}}{\partial x} \quad (11)$$

where we assume the direction of current x -direction ($\mathbf{j}_e = j_e \mathbf{e}_x$), $b_J = P j_e \mu_B / e M_s (1 + \xi^2)$, and $c_J = P j_e \mu_B \xi / e M_s (1 + \xi^2)$. Note that b_J and c_J have the unit of velocity. The “ b_J ” term has been already proposed by Bazaliy et al. [12] when they consider a ballistic motion of conduction electrons in the half-metal materials. Recently Tatara and Kohno also derived similar expression [8]. We have seen that this term describes the adiabatic process of the non-equilibrium conduction electrons. The “ c_J ” term is completely new; it is related to the spatial mis-tracking of spins between conduction electrons and local magnetization. While this term is known in the physics of domain wall resistance [14, 15, 16], it also gives rise a non-adiabatic spin torque, the last term in Eq. (11). At first sight, one might think that this “ c_J ” term may be discarded since it is much smaller than the “ b_J ” term ($c_J/b_J = \xi \approx 10^{-2}$). We will show below that the terminal velocity of a domain wall is *independent* of the strength of “ b_J ”, rather it is controlled by this small “ c_J ” term. Thus, experimental analysis on the domain wall motion must include this new “ c_J ” term.

To make a concrete prediction on the domain wall dynamics from Eq. (11), we consider a Néel wall in a magnetic nanowire whose magnetization vector only depends on the position

along the wire, i.e., $\mathbf{M} = \mathbf{M}(x, t)$. The effective field entering Eq. (11) is modeled by

$$\mathbf{H}_{eff} = \frac{H_K M_x}{M_s} \mathbf{e}_x + \frac{2A}{M_s^2} \nabla^2 \mathbf{M} - 4\pi M_z \mathbf{e}_z + H_{ext} \mathbf{e}_x \quad (12)$$

where H_K is the anisotropy field, A is the exchange constant, and $4\pi M_z$ is the demagnetization field. In the presence of the spin torque, we follow the Walker's prescription of the domain wall motion by introducing a trial function $\mathbf{M}(\theta, \varphi)$ where (θ, ϕ) are polar angles in the following form [17],

$$\varphi = \varphi(t); \quad \ln \tan \frac{\pi - \theta}{2} = \frac{1}{W(t)} \left(x - \int_0^t v(\tau) d\tau \right) \quad (13)$$

The first equation assumes that the projection of the magnetization vector in the domain wall on the yz plane is independent of the position. The second equation in Eq. (13) postulates that the domain wall shape remains a standard Néel-wall form except that the wall width $W(t)$ varies with time and the wall moves at velocity $v(t)$. By placing Eqs. (13) and (12) into Eq. (11), and by assuming the domain wall width changes slowly as in the Walker's theory, we can find two coupled differential equations for determining the domain wall distortion parameters $\varphi(t)$ and $W(t)$. Interestingly, the expression for the velocity of the domain wall at the initial application of the current is [18]

$$v(0) = -\frac{1}{1 + \alpha^2} \left(\frac{\alpha \gamma H_{ext}}{W(0)} + b_J + \alpha c_J \right) \quad (14)$$

while the terminal velocity of the domain wall is

$$v_T \equiv v(\infty) = -\frac{\gamma H_{ext}}{W(\infty) \alpha} - \frac{c_J}{\alpha}. \quad (15)$$

where $W(\infty)$ is the terminal wall width that is slightly smaller than the initial Néel wall width $W(0)$. Equations (14) and (15) reveal the different roles played by the adiabatic (b_J term) and non-adiabatic (c_J term) spin torques: the adiabatic torque is most important at the initial motion of the wall while the non-adiabatic c_J controls the terminal velocity of the domain wall. The adiabatic torque causes the domain wall distortion. The distorted domain wall is able to completely absorb the adiabatic spin angular momentum so that the net effect of the adiabatic torque on the domain wall velocity becomes null, i.e., domain wall stops. In contrast, the non-adiabatic spin torque behaves as a non-uniform magnetic field $c_J \partial \mathbf{M} / \partial x$ that can sustain a steady state wall motion. Although the magnitude of the non-adiabatic torque c_J is about two orders of magnitude smaller than adiabatic torque b_J ,

the terminal velocity is inversely proportional to the damping parameter which makes the velocity comparable to b_J .

Finally, we emphasize that the present study has resolved an outstanding mystery between the recent experimental observation [19] and the theoretical prediction based on the adiabatic spin torque. It has been recognized that a critical current density of the order of $10^9 - 10^{10} \text{ A/cm}^2$ is required to move a perfect domain wall [8, 13] if we only use the adiabatic spin torque b_J . Experimentally, a velocity about 3 m/s was observed in a NiFe nanowire when a current density $1.2 \times 10^8 \text{ A/cm}^2$ was applied. This velocity had been assumed to relate with b_J [19] in spite of the apparent *qualitative and quantitative disagreement* between theory and experiment. Here, we have pointed out that b_J is simply an initial velocity of the domain wall and the measured velocity was the terminal velocity. For the experimental current density of $1.2 \times 10^8 \text{ A/cm}^2$, the adiabatic spin torque alone is unable to sustain a constant velocity. By including a small non-adiabatic torque c_J , we find the domain wall velocity is now c_J/α in the absence of the magnetic field, see Eq. (15). Although the numerical values of both the exchange constant J_{ex} and the damping parameter α are not precisely known in ferromagnets, we estimate that the wall velocity should be $6 \sim 60$ (m/s) for the above current density if we use the parameters indicated before (taking $\alpha = 0.01 \sim 0.1$ for permalloy). While the experimental velocity is smaller than our estimated value, it is reasonable that we do not include any defects that may reduce the observed velocity significantly.

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- [1] M. N. Baibich *et al.*, Phys. Rev. Lett. **61**, 2472 (1998).
 - [2] L. Berger, Phys. Rev. B **54**, 9353 (1996); J. Slonczewski, J. Magn. Magn. Mater. **159**, L1 (1996).
 - [3] Y. Tserkovnyak, A. Brataas, and G. E. W. Bauer, Phys. Rev. Lett. **88**, 117601(2002).
 - [4] R. Urban, G. Woltersdorf, and B. Heinrich, Phys. Rev. Lett. **87**, 217204 (2001).
 - [5] J. Ho, F. C. Khanna, and B. C. Choi, Phys. Rev. Lett. **92**, 097601 (2004).
 - [6] B. Heinrich *et al.*, Phys. Rev. Lett. **90**, 187601 (2003); E. Simanek and B. Heinrich, Phys. Rev. B **67**, 144418 (2003).
 - [7] A. Brataas *et al.*, Phys. Rev. Lett., **91**, 166601 (2003).

- [8] G. Tatara and H. Kohno, Phys. Rev. Lett., **92**, 086601 (2004).
- [9] Equation (7) may be written in the form of $\nabla^2 y - (1/\lambda^2)y = f(\mathbf{r})$ where $y = \delta m_x \pm i\delta m_y$ is the spin density in the rotating frame, δm_x and δm_y are two transverse components of $\delta \mathbf{m}$ (perpendicular to $\mathbf{M}(\mathbf{r})$), and $\lambda = \sqrt{D_0(1/\tau_{sf} + i/\tau_{ex})^{-1}}$. The solution of the above differential equation is $y(\mathbf{r}) = \int \frac{\exp(-|\mathbf{r}-\mathbf{r}'|/\lambda)}{4\pi|\mathbf{r}-\mathbf{r}'|} f(\mathbf{r}') d^3\mathbf{r}'$. For a slowly varying function $f(\mathbf{r}')$, one may replace $f(\mathbf{r}')$ by $f(\mathbf{r})$ in the integral and thus $y = -\lambda^2 f(\mathbf{r})$, i.e., one can neglect the first term of Eq. (7).
- [10] Y. Tserkovnyak, G. A. Fiete, and B. I. Halperin, arXiv:cond-mat/0403224.
- [11] V. Kambersky, Can. J. Phys. **48**, 2906 (1970).
- [12] Ya. B. Bazaliy, B. A. Jones, and S.-C. Zhang, Phys. Rev. B **57**, R3213 (1998).
- [13] Z. Li and S. Zhang, to appear in Phys. Rev. B (2004).
- [14] P. M. Levy and S. Zhang, Phys. Rev. Lett. **79**, 5110 (1997).
- [15] G. Tatara and H. Fukuyama, Phys. Rev. Lett. **78**, 3773 (1997).
- [16] E. Simanek, Phys. Rev. B **63**, 224412 (2001).
- [17] N. L. Schryer and L. R. Walker, J. Appl. Phys. **45**, 5406 (1974).
- [18] The Walker's trial function breaks-down at a very high current density at least $10^9 A/cm^2$, see Ref. [8] and [13]. Here we assume the current is smaller than $10^9 A/cm^2$.
- [19] A. Yamaguchi *et al.*, Phys. Rev. Lett. **92**, 077205 (2004).